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Computer Vision

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Laplacian of Gaussian (LoG)
Gaussian Smoothing Filter

- To smooth an image, convolve (or correlate) the filter with the image
- Attenuates high frequencies

\[ h(x, y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/(2\sigma^2)} \]
Sharpening Spatial Filters

• First derivative (can also do central difference)
  \[
  \frac{\partial f}{\partial x} \approx f(x+1) - f(x)
  \]

• Second derivative
  \[
  \frac{\partial^2 f}{\partial x^2} \approx f(x+1) - 2f(x) + f(x-1)
  \]

• Laplacian
  \[
  \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \approx f(x+1, y) - 2f(x, y) + f(x-1, y) \\
  + f(x, y+1) - 2f(x, y) + f(x, y-1)
  \]
Laplacian of a Gaussian (LoG)

\[ \nabla^2 g \]

\[ \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/(2\sigma^2)} \]

- Convolve with a Gaussian, then take the Laplacian; or do the equivalent operation

\[ \nabla^2 (g \ast I) = \left( \nabla^2 g \right) \ast I \]
Laplacian of a Gaussian

- The operator’s scale is determined by $\sigma$

$$\nabla^2 g(x, y) = -\frac{1}{\pi \sigma^4} \left( 1 - \frac{r^2}{2\sigma^2} \right) e^{-r^2/(2\sigma^2)}$$

- where $r^2 = x^2 + y^2$ (it is circularly symmetric)

![Diagram of Laplacian of a Gaussian](image)
FIGURE 10.14
Laplacian of a Gaussian (LoG).
(a) 3-D plot.
(b) Image (black is negative, gray is the zero plane, and white is positive).
(c) Cross section showing zero crossings.
(d) $5 \times 5$ mask approximation to the shape of (a).

\[
\begin{array}{cccccc}
0 & 0 & -1 & 0 & 0 \\
0 & -1 & -2 & -1 & 0 \\
-1 & -2 & 16 & -2 & -1 \\
0 & -1 & -2 & -1 & 0 \\
0 & 0 & -1 & 0 & 0 \\
\end{array}
\]
Example

\[
\sigma = 2.0; \\
h = \text{fspecial}'log', 6*\sigma, \sigma) ; \\
I = \text{imread}'cameraman.tif' ; \\
\text{figure, imshow(I, [])} \\
I = \text{double}(I) ; \\
I2= \text{imfilter}(I,h) ; \\
\text{figure, imshow(I2, [])}
\]

- LoG acts like a band pass filter
  - Constant intensity regions -> zero response
  - Very small image features -> low response
- It gives the strongest response where the image features are about the same size as the filter
- Acts like a “blob” detector
Other Properties of LoG

• LoG can be approximated by a difference of Gaussians

\[ \nabla^2 g(\sigma) \approx g(\sigma_1) - g(\sigma_2), \quad \sigma_2 / \sigma_1 \approx 1.6 \]

• There is evidence that the human visual system does edge detection similar to LoG
  – Excitatory center, inhibitory surround
  – A set of spatial frequency tuned channels